PHYS 320 ANALYTICAL MECHANICS

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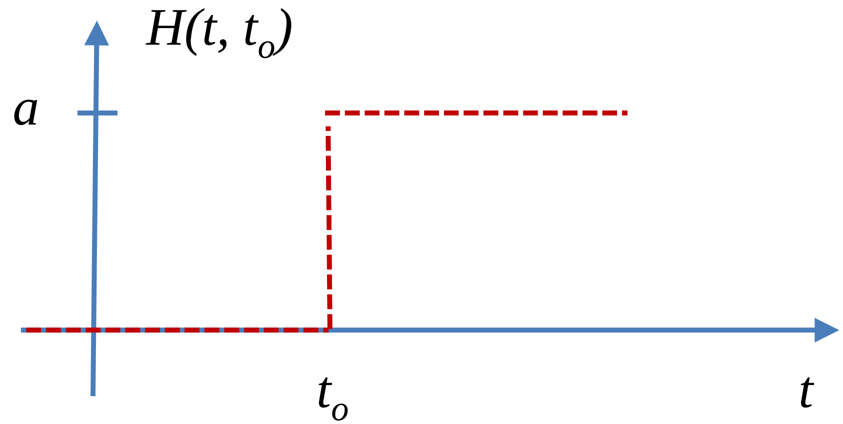
Driven damped oscillator: response to step/impulse

The differential equation $m\ddot{x} + c\dot{x} + kx = F_{ext}(t)$

Has transient and steady-state solutions $x(t) = x_c(t) + x_p(t)$

Step function force:

- If $F_{ext}(t) = F_o\, H(t, t_o)$, $H(t,t_o) = \begin{cases} 0, & t < t_o \\ a, & t > t_o \end{cases}$



Solutions are of the form: $x(t) = \dfrac{H(t,t_o)}{a}\left\{ e^{-\gamma(t-t_o)}[\mathrm{A}_1\cos(\omega_d(\mathrm{t}-\mathrm{t}_o)) + \mathrm{A}_2\sin(\omega_d(\mathrm{t}-\mathrm{t}_o))] + \dfrac{a}{\omega_o^2} \right\}$

Driven damped oscillator: response to step/impulse

Step function force:

If initial conditions are such that $x(t_o) = 0$ and $dx(t_o)/dt = 0$, then

$$x(t=t_o) = \left[e^{-\gamma(t-t_o)}[\mathrm{A}_1\cos(\omega_d(\mathrm{t}-\mathrm{t}_o)) + \mathrm{A}_2\sin(\omega_d(\mathrm{t}-\mathrm{t}_o))] + \frac{a}{\omega_o^2} \right|_{t=t_o} = 0$$

$$\left.\frac{dx(t)}{dt}\right|_{t=t_o} = 0$$

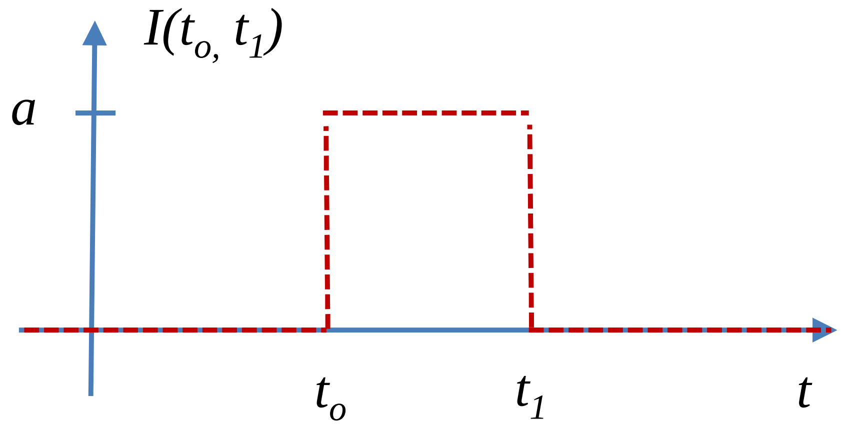
$$A_1 = -\frac{a}{\omega_o^2} \qquad A_2 = -\frac{\gamma a}{\omega_1 \omega_o^2}$$

Driven damped oscillator: response to step/impulse

Impulse function force:

- If $F_{ext}(t) = F_o\, I(t_o,t_1\,)$,

$$I(t_o,t_1) = H(t_o) - H(t_1) = \begin{cases} 0, & t < t_o \\ a, & t_o < t < t_1 \\ 0, & t > t_1 \end{cases}$$



Solutions are of the form: (for $t > t_1$)

$$x(t) = \left\{ \frac{a}{\omega_o^2} - e^{-\gamma(t-t_o)}\left[\frac{a}{\omega_o^2}\cos(\omega_d(\mathrm{t}-\mathrm{t}_o)) + \frac{\gamma a}{\omega_1\omega_o^2}\sin(\omega_d(\mathrm{t}-\mathrm{t}_o))\right] \right\}$$

$$- \left\{ \frac{a}{\omega_o^2} - e^{-\gamma(t-t_o-\tau)}\left[\frac{a}{\omega_o^2}\cos(\omega_d(\mathrm{t}-\mathrm{t}_o-\tau)) + \frac{\gamma a}{\omega_1\omega_o^2}\sin(\omega_d(\mathrm{t}-\mathrm{t}_o-\tau))\right] \right\}$$

Hamilton's Principle

- *Of all the possible paths along which a dynamical system may move from one point to another within a specified time interval (consistent with any constraints), the actual path followed is that which minimizes the time integral of the difference between the kinetic and potential energies.*

Published in two papers, 1834, 1835

Calculus of Variations

$$J = \int_{x_1}^{x_2} f\,(y(x), y'(x); x)\,dx$$

$$where \quad y'(x) \equiv dy/dx$$

Neighboring function: (*parametric representation*)

$y(\alpha,x) = y(0,x) + \alpha\,\eta(x) = y(x) + \alpha\,\eta(x)$

If $\alpha = 0$, then $y(0,x) = y(0,x) = y(x)$ is the function that yields the extreme value in J

Has extreme values (is "stationary"*) when*

$$\left.\frac{\partial J}{\partial \alpha}\right|_{\alpha=0} = 0 \quad \forall\, \eta(x)$$

$$\frac{\partial f}{\partial y} - \frac{d}{dx}\left(\frac{\partial f}{\partial y'}\right) = 0$$

Euler-Lagrange Equation